AP Calculus AB/BC Summer Packet

1) Find the x and y-intercepts for each of the following:
   a) \( y = x^2 + x - 1 \)                     b) \( y = (x - 1)\sqrt{9 - x^2} \)
   c) \( y = \frac{x^2+3x}{(3x+1)^2} \)            d) \( x^2 y - x^2 + 4y = 0 \)

2) Find all points of intersection of each of the following:
   a) \( 2x - 3y = 13 \)  and  \( 5x + 3y = 1 \)   b) \( x^2 + y^2 = 25 \)  and  \( 2x + y = 10 \)
   c) \( y = x^3 - 4x \)  and  \( y = -x \)        d) \( y = x^4 - 2x^2 + 1 \)  and  \( y = 1 - x^2 \)

3) Write the equation of the line with the following characteristics
   a) Passes through \((3, -4)\) and \((5,2)\)
   b) Passes through \((-1,2)\) and \((2,8)\)
   c) Passes through \((2,5)\) and has a slope of \(-4\).
   d) Passes through \((6, -1)\) and has a slope of \(\frac{1}{2}\).

4) For the function \( f(x) = x^2 - 4x \), find each of the following:
   a) \( f(4) \)    b) \( f\left(\frac{3}{2}\right) \)    c) \( f(q) \)    d) \( f(t + 4) \)

5) Find the value of \( \frac{f(x) - f(3)}{x - 3} \) for each of the following functions:
   a) \( f(x) = 3x + 7 \)                     b) \( f(x) = 3x^2 - 2x + 1 \)          c) \( f(x) = \frac{6}{x} \)

6) Find the value of \( \frac{f(x+h)-f(x)}{h} \) for each of the following functions:
   a) \( f(x) = 3x + 7 \)                     b) \( f(x) = 3x^2 - 2x + 1 \)          c) \( f(x) = \frac{6}{x} \)
7) For the piecewise function \( p(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases} \) find each of the following:

a) \( p(-3) \)  

b) \( p(0) \)  

c) \( p(5) \)  

d) \( p(10) \)

8) Find the inverse of each of the following:

a) \( f(x) = 4x - 3 \)  

b) \( g(x) = \frac{2x + 3}{x - 4} \)  

c) \( h(x) = x^3 + 1 \)

9) For each of the following, find \( f(-x) \) and use it to determine if the function is odd, even, or neither:

a) \( f(x) = x^2(4 - x^2) \)  

b) \( f(x) = \sqrt[3]{x} \)  

c) \( f(x) = x \cos x \)

10) Graph each of the following:

a) \( f(x) = -3x + 2 \)  

b) \( f(x) = 1 - x^2 \)  

c) \( f(x) = \begin{cases} 2x - 3, & x \leq 3 \\ (x - 6)^2 - 2, & x > 3 \end{cases} \)

d) \( f(x) = |6 - x| \)  

e) \( f(x) = \sqrt{9 - x^2} \)

11) State the value of all 6 trig functions for each of the following angles:

a) \( \frac{5\pi}{4} \)  

b) \( \frac{11\pi}{6} \)  

c) \( \frac{14\pi}{3} \)  

d) \( \frac{\pi}{2} \)  

e) \( \pi \)

12) Simplify the following:

a) \( \sin x \cos x \tan x \sec x \csc x \)  

b) \( \frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x} \)

c) \( \csc x - \cos x \cot x \)  

d) \( \sec \theta \sin \theta \cot \theta \csc \theta \)

13) Rewrite each of the following as an algebraic expression with no trig functions involved. (Hint: draw triangles and use Pythagorean Theorem.)

a) \( \sin \text{arccos} 2x \)  

b) \( \cot \text{arcsin} x \)  

c) \( \sin \text{arctan} 3x \)
"BA" (BAD ALGEBRA) SECTION – The solution to each of the following equations contains at least one step (and possibly more) that involves bad algebra. Your job is to find the bad algebra, explain (very briefly) why it is bad algebra, and re-solve the problem correctly. (All BA’s in ap calculus result in full credit lost for any problem, every time they occur.)

a) \(10x^2 + 7x = 12\)
   \[x(10x + 7) = 12\]
   \[x = 12 \text{ and } 10x + 7 = 12\]
   \[x = 12, x = \frac{1}{2}\]

b) \((x - 5)^2 = 16\)
   \[x^2 - 25 = 16\]
   \[x^2 = 41\]
   \[x = \pm\sqrt{41}\]

c) \(\sin 2x = \sin x, \ 0 \leq x < 2\pi\)
   \[2 \sin x = \sin x\]
   \[\sin x = 0\]
   \[x = 0\]

d) \(x^3 = x^2\)
   divide both sides by \(x^2\)
   \[x = 1\]

e) \(\sqrt{x^2 - 16} = 3\)
   \[x - 4 = 3\]
   \[x = 7\]

f) \(x = \sqrt{4}\)
   \[x = \pm 2\]

g) \(\sqrt{x} = 9\)
   \[x = 3\]